# COMPARISION OF NUMERICAL METHODS USED IN THE LIGHT SYSTEMS 

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#### Abstract

The paper deals with numerical methods, which are used in the light systems calculations. There are the principles, advantages and disadvantages of this method. The main part of the paper is description of two numerical models. The first of them was built in the Ansys system. The second of them was built in the MATLAB by the Ray-tracing method. Both models were verified by experiments and results were compared.


## 1. INTRODUCTION

For the numerical modeling of the light systems were used this numerical method: Raytracing, Radiosity, Flow method, Point method and Elementary transformation method. The Ray - tracing and the Radiosity methods are most widely used methods in the light systems. We can get realistic projection with this method. These methods are often used for 3D scenes projections and lighting calculation, enable modeling indirect lighting, shadows and color transition, by this we can manage realistic projection of the scene. The Flow method, the Point method and the Elementary transformation method are intended for lighting calculation. The Elementary transformation method is used for lighting calculation from a reflector or for calculation of light source with reflector based on requirement on surface light. The Flow and Point methods are intend to lighting calculation of scenes based on initial conditions. In this paper are described the Ray-tracing and the Radiosity methods which are used for creation of the numerical model.

## 2. DESCRIPTION OF USED METHODS

### 2.1. RAY-TRACING

Principle of the Ray-tracing method is searching of light rays which come through scene and falls on screen or into eye. We don't proceed from light sources but from screen pixels in search of the rays. This ray originates by sum of fragmental (secondary) rays, which originate by reflection and refraction at a point of intersection ray with objects and shadow rays from light sources in the space of scene. We create a point of intersection tree from rays originate on refraction with objects. The light model is computed in all point of intersection. The resultant ray is given by a sum of separate rays from point of intersection tree
from top of tree up to root. Demonstration of the scene and point of intersection tree is shown in fig. 1. Figures are assumed from [1]. Tracked ray is drawn by solid line and shadow ray by a dash line. Shadow ray indicates which light sources are directly share in lighting of point of intersection and which in point of intersection drop shadow over object on the scene.


Figure 1 Demonstration of scene, point of intersection tree
Light model is compute by next equation:

$$
\begin{equation*}
I_{v}=I_{s}+I_{d}+I_{a}+I_{r}+I_{t} \tag{1}
\end{equation*}
$$

where $I_{s}$ is component of reflection, $I_{d}$ is diffuse component, $I a$ is component of surround relevant only for light sources, $I_{r}$ represent reflected or refract ray and $I_{t}$ is intensity of ray on crossing through the transparent object.

### 2.2. Radiosity

This method is called radiating method. Radiosity appears from hypothesis, that the scene is closed to power and containing all needed light sources. Surface is divided to n plane facets. These facets prove receive, radiate and reflect light energy. We find out physical value called Radiosity B on these facets. Radiosity is representing total energy de-excitation from surface to time unit. This energy is equal sum of emitted and reflected energy. Radiosity on separate facets is constant. All surfaces are considered as emitter, surfaces can emitted own energy or only reflect energy. Power transmission from surfaces in scene is described by equation:

$$
\begin{equation*}
B_{i}=E_{i}+\rho_{i} \sum_{i=1}^{n} B_{j} F_{i j} \tag{2}
\end{equation*}
$$

where $B_{i}$ is Radiosity of surface $P_{i}, E_{i}$ is own emissivity of surface $P_{i}, F_{i}$ is reflectivity of surface, $F d A_{j} d A_{i}$ is form factor and $\rho$ is radiant reflectivity. Equation (2) is called basic Radiosity equation. If we use this equation for all $n$ surfaces, we are getting system of $n$ equations with unknown $B_{i}$. Valuables of $E_{i}$ are non - zero only for light sources. Form factor describes cross power exchange between surfaces. Form factor $F_{i j}$ describes, what part of energy is emitted of surface $P_{i}$ land on surface $P_{j}$. Value of form factor is depending only on geometry of surfaces.

$$
\begin{equation*}
F_{i j}=F_{A_{i} A_{j}}=\frac{1}{A_{i}} \int_{A_{i} A_{j}} \int^{\cos \varphi_{i} \cos \varphi_{j}} \frac{\pi r^{2}}{} V_{i j} d A_{j} d A_{i} \tag{3}
\end{equation*}
$$

where $A_{i}$ is area of surface, $r\left(\varphi_{i}, \varphi_{j}\right)$ is distance (angel) between surfaces.

## 3. RAY- TRACING NUMERICAL MODEL

MATLAB software was chosen for creation of numerical model of the Ray - tracing method. Numerical model was created along the table lamp, its dimensions are on fig. 2. Numerical model was simplified against physical model. Model was created in vertical section, this simplification we can use because table lamp is rotational symmetrical. Light bulb was replaced by omnidirectional light source. The top hemisphere part of shield was ignored and replaced by abscissa EF. The upper part of shield was filled by dark dead color.

### 3.1. DESCRIPTION OF ALGORITHM

There were engaged points by x ordinate and y ordinate of these points, from this coordinates was created common equations of this bisectors. This equation was modified to possibility of coordinates of points substitution. For points A, B is the common equation in form:

$$
\begin{equation*}
\left(b_{2}-a_{2}\right) x-\left(b_{1}-a_{1}\right) y-\left[\left(b_{2}-a_{2}\right) a_{1}-\left(b_{1}-a_{1}\right) a_{2}\right]=0 \tag{4}
\end{equation*}
$$

Where $a_{1}, b_{1}$ are $x$ coordinates of point and $a_{2}, b_{2}$ are $y$ coordinates of point.
There was set radiant reflectivity on abscissas. On abscissas P1, P2a, P2b and P3 was set radiant reflectivity to 0,7 and abscissa $P 2 c$ to value 0 . The illuminance of one ray was calculated by:

$$
\begin{equation*}
E_{p}=\frac{\phi_{z} / A_{z}}{N_{p}} \tag{5}
\end{equation*}
$$

where $E_{p}$ is illuminance of one ray, $\phi_{z}$ is luminous flux of light bulb, $A_{z}$ is area of light bulb and $N_{p}$ is number of raies.


Figure 2 Dimension of model, distribution of illuminance
In the main part of algorithm is broadcast of 3600 rays with angle $0.01^{\circ}$. There are tested points of intersection of all rays with defined bisectors. If it is found the point of intersection, there was calculated angle of reflection and illuminance of ray is lowered by radiant reflectivity. Following of ray is finished when number of rays is larger than 20 or when ray lands to bisector Pp or P 2 c . Interpretation of the illuminance is performed on bisector Pp in interval $\langle-1 \mathrm{~m} ; 1 \mathrm{~m}\rangle$. In this interval are values of the illuminance summed in 5 mm intervals. The results of the numerical model are in fig. 2 (right).

## 4. NUMERICAL MODEL OF THE RADIOSITY

ANSYS software was chosen for creation of the numerical model of the Radiosity method. ANSYS is using Radiosity method for calculation of thermal transmittance by radiating. We can use this method for solving light tasks thanks to analogy between thermal and light field. The light source with illuminance $E_{s}(\mathrm{~lx})$ is accordant with density of heat flux $q^{\prime \prime}$ and luminous flux $\phi(\operatorname{lm})$ is accordant with heat flow $q^{\prime}$. The resultant luminous flux is given by equation (6), described in [2], [3].

$$
\begin{equation*}
\Phi_{e}=\frac{T_{f, e}}{S_{n, e}} \tag{6}
\end{equation*}
$$

where $\Phi_{e}$ is luminous flux on a element, $T_{f, e}$ is heat flow of element and $S_{n, e}$ is area of element. According to Stefan- Boltzmann law of heat transfer between surfaces with indexes $i, j$, radiating described

$$
\begin{equation*}
q_{r i}=\sigma \varepsilon_{i} A_{i} S_{i}\left(T_{i}^{4}-T_{j}^{4}\right), \text { na } \Gamma_{T}, \tag{7}
\end{equation*}
$$

where $\Gamma_{T}$ is bound of $\Omega_{T}, q_{r i}$ is specific heat of transferred from surface with index $i, \sigma$ is Stefan-Boltzmann constant, $\varepsilon_{i}$ emissivity of surface, $A_{\mathrm{i}, \mathrm{j}}$ is factor of projection, $S_{i}$ is area of surface $i, T_{i}, T_{j}$ are temperature of surfaces $i, j$.


Figure 3 Dimensions of model, geometric model ANSYS


Figure 4 Distribution of heat flow and illuminance

2D geometric model was created regarding to rotating symmetry. Dimensions of numerical model are shown in fig. 3 together with the model in ANSYS. To create a mesh was chosen element PLANE77. For material model was set heat conductivity $\mathrm{k}=10^{-6} \mathrm{~W} / \mathrm{m} . \mathrm{K}^{-1}$, emissivity (determinates how many energy is radiated and how many is absorbed) on side of shield is 0.7 on top shield and socket 0.2 , on light bulb 0.9 and on monitored surface 0.1 . Heat flow 450 W was set on light bulb. The results of numerical model are shown in fig. 4.

## 5. MEASUREMENT VERIFICATION

Both models were verificated by measurement. Measurement was proceeded in dark chamber with stable luminous flux flow. Light characteristic was measured in $1 / 4$ (rotating symmetry) in measuring points distanced 0.1 m . Measuring points were organized in square mesh. Light source was bulb $40 \mathrm{~W} / 230 \mathrm{~V}, 450 \mathrm{~lm}$. For measurement was used illuminometer BEHA UNITEST 93514. Measured values in x axis are in table 1.
Table 1 Measured values

| $1[\mathrm{~mm}]$ | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{E}[1 \mathrm{x}]$ | 442 | 387 | 292 | 194 | 111 | 47 | 21 | 12 |

## 6. CONCLUSION

Comparison of results both numerical models and results form measurement are in Fig. 5. We can see that the results of Radiosity and measured results are identical. Confrontation of the Ray-tracing results and measured results are biggest deviation is on distance 0.4 m that is due to sharp transients between light and shadow in Ray-tracing method. After confrontation results we can see that both methods can achieve highly exact results.


Figure 5 Confrontation of results

## REFERENCES

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